A Game Theory Analysis of the Mechanisms of Suburbanization and its Effects on Social Capital

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1 Introduction

From the window of an airplane, North American development patterns take a familiar, seemingly uniform form. Sprawling suburbs, expansive metropolitan areas, and seas of parking lots define the American landscape. A sizable majority of Americans and wealthy American neighborhoods reside quite far from the city center in suburban or exurban developments designed car-dependence (Parker et al, 2018). Among wealthy and white Americans, who historically have had the highest degree of freedom in choosing where to reside, the majority is even more pronounced (Parker et al, 2018). Curiously, though, and surprising to many Americans, this development pattern is not the norm historically or internationally. Upon traveling to other countries, Americans are faced with far denser urban centers with often a greater diversity of accommodations—far from the enforced uniformity of single-family homes found in much of North America (Angel et al., 2010). In the United States itself, this suburbanization is a relatively recent trend, with merely 13 percent of Americans residing in suburbs prior to the Second World War (Nicolaides et al., 2017).

So, what changed? Following the war, veterans returned to massive housing shortages, made worse by the contemporary baby boom dramatically increasing the American demand for housing. At the same time, mass adoption of affordable automobiles, easy access to capital in the form of a Federal Housing Authority loan, massive public investment in highways, and a desire for an antidote for often crowded, dirty American cities converged to create the model of the modern American suburb (Nicolaides et al., 2017).

This suburban expansion was quick, too, with the number of annual housing starts going from about 140,000 in the 1940s to more than ten times that amount ten years later (Nicolaides et al., 2017). However, this rapid move from one extreme of density to another was not without long-term consequences. As early as 1950, concerns were emerging about the impact of this mass movement on cities, especially regarding the city budget (Laas, 1950). When a city's tax base dramatically decreases while demands on infrastructure dramatically increase from the increase in commuters, a greater burden is placed on the now smaller population which remains in the city (Baum-Snow, 2007). Additionally, an increase in motor vehicle usage within the city has led to a number of ill effects on the residents who remained. These include but are not limited to: significant health complications resulting from prolonged exposure to high levels of air pollutants, chronic stress and health complications resulting from constant, prolonged exposure to road noise, deadlier roads, decreased usable space, all combined with the lack of resources to deal with these problems (Zhang et al.; see also IIHS; Münzel et al., 2018; Tobollik et al., 2019) . Additionally, with a lack of funding and insufficient density to support affordable public transportation, streetcars, bikes, and buses fell out of favor, replaced by the individual car, further aggravating all the aforementioned problems as city residents also need to drive everywhere (Norton, 2014). In fact, in the United States, 63 percent of all trips by car are less than 5 miles, with 28 percent less than 1 mile (NHTS, 2009). In short, it is difficult to argue that suburbanization and white flight made the American city a nicer place to live.

While the effects of suburbanization on those who remained in the cities are quite clear, the long-term effects on those living in these newly constructed suburbs is a bit more subtle. With the creation of developments with urban amenities at near-rural density, the per-resident cost of creating and maintaining roads, plumbing, sewer, electricity, garbage collection, and other amenities is dramatically increased, especially for the predominant single-family home. While the easy access to capital and the post-war economic boom made paying for the initial infrastructure cost manageable for private developers, maintenance would be publicly funded as it was generally handed over to the municipality after completion (Mahron Jr., 2013). Given that the average life cycle for replaceable infrastructure such as roads and sewer lines is about 25 years, for the American model of suburban development to be financially sustainable, one of two assumptions must be met—either the tax revenue from new growth exceeds long term maintenance and replacement costs or the development will perpetually grow at an ever-accelerating rate (Norton, 2014). Given that tax incentives are a popular draw for suburban communities, it is no wonder that, in the face of slowing growth, many American municipalities are struggling with insolvency (Norton, 2014).

Another potential, yet more subtle, effect of suburbanization on suburbanites could be an increase in loneliness and a loss of human capital. Over recent decades, the share of Americans exhibiting traits of loneliness has increased dramatically, with each generation getting progressively lonelier than their parents (Newswire, 2018). While some have placed the blame squarely on social media and the rise of ever-present digital entertainment, the progressive increase in loneliness traces back to generations reared long before these forms of media were even invented (Newswire, 2018). In Bowling Alone, Putnam famously asserts that social capital and community have been in precipitous decline since at least the 1960s (Putnam, 1999). One proposed mechanism to explain this phenomenon is that the physical separation created by the car-dependency and sparse, cul-de-saced housing of suburbanization have reduced the aggregate level of interaction, causing the deterioration of social networks.

2 Approach and Overview of Methods

In this paper, we propose a model explaining this suburbanization process along with an associated decline in utility resulting from increased transportation costs and the aforementioned decrease in interaction levels as the outcome of a network game in equilibrium. Building on the framework of Helsley and Zhou, we introduce congestion costs as an exacerbating factor, further decreasing the utility resulting from the move to higher levels of "suburbanization" found in equilibrium. Finally, we will propose a Pigouvian "congestion tax" which would alter the utility function so as to maximize total aggregate utility in equilibrium.

While Helsley and Zhou find that there is an endogenous spatial stratification associated with an exogenous social stratification, we are more interested in exploring a mechanism by which self-interested individuals would largely choose to move to a periphery area as well as showing that there exists an alternative equilibrium which is strictly better for all individuals in the game (Helsley et al., 2014). To do this, we introduce a two-stage game. However, first, we must establish a base model from which to work. In the base model, agents with exogenous location choose their level of interaction with the other players. The possible locations consist of two nodes: the center, which can be thought of as the city, and the periphery, which can be thought of as the suburbs. Given an exogenous network of social connections, agents can choose how often they visit the center, maximizing on a utility function dependent on transportation costs (including congestion), how many times an agent visits the interaction center, and how many times an agent's connections visit the interaction center.

In the two stage, we assume all agents are rational actors making choices strictly in their own self-interest. All agents are identical, face the same utility function, and know this fact. Equally, the social network is transparent to all agents. In the first stage, agents can choose their location within the network. In the second, agents choose the optimal number of visits to the interaction center according to the base model and subutility function.

3 Part 3: Base Model

To begin, we assign n agents to a both a social and geographic network. The geographic network consists of two nodes, or locations: the *center* and the *periphery*. All agents must be in one of these locations. We assign x_i to the location and distance from the center of agent i and, normalized to the values $x_i \in \{0, 1\}$, $\forall_i = 1, 2, ..., n$, with 1 indicating location of agent i at the periphery and 0 indicating location of agent i at the center. To begin, we consider this location exogenous, but will consider location choice in a later section. The social network g is a set of initially identical agents $N = \{1, ..., n\}$, $n \geq 2$, and a set of edges indicating connections between. Our adjacency matrix $G = [g_{ij}]$ indicates all direct connections in the social network, such that agent i is directly connected to agent j if and only if $g_{ij} = 1$ while $g_{ij} = 0$ otherwise. By convention, $g_{ii} = 0$. Thus, G is a square matrix with zeros on its diagonal. The neighbors of agent i in the social network are denoted by N_i where $N_i = \{allj|g_{ij} = 1\}$. The degree of node agent i in the social network is thus denoted by $d_i = |N_i|$. The initial preference function of agent i is denoted by:

$$U_i(v_i, v_{-i}, g) = z_i + u_i(v_i, v_{-i}, g)$$

where v_i is the number of visits agent i makes to the center, v_{-i} is a vector of the number of visits each other agent makes, and $u_i(v_i, v_{-i}, g)$ represents the subutility function for the utility resulting from visits to the center. This subutility function takes a linear quadratic form and can be represented as:

$$u_i(v_i, v_{-i}, g) = \alpha v_i + \frac{1}{2}v_i^2 + \theta \sum_{j=1}^{n} g_{ij} v_i v_j$$

where parameters $\alpha > 0$ and $\theta > 0$. Here, α represents the value of each interaction to agent i and θ is a parameter for the intensity of interactions. We assume that each visit to the center results in exactly one interaction. Agents also derive utility from good z, where the budget balance for z_i is written as:

$$z_i = y - t x_i v_i \sum_{j=1}^{n} x_j$$

Here, y represents income and t represents transportation cost to the center. Again, location of agent i is represented by the binary x_i, meaning transport cost is only considered for those living in the periphery. Congestion cost is also considered, with $\sum_{j=1}^{n} x_j$ linearly increasing transport cost as the number of other agents who live in the periphery increases. For all practical purposes, this congestion function will be attached to the base transportation cost t, jointly representing the total transportation cost of agent i taking one visit to the center when $x_i = 1$. Combining z_i with the subutility function, we obtain

$$U_i(v_i, v_{-i}, g) = y + \alpha_i v_i - \frac{1}{2}v_i^2 + \theta \sum_{j=1}^{n} g_{ij} v_i v_j$$

where $\alpha_i = \alpha - t x_i \sum_{j=1}^{n} x_j$. We assume that $\alpha_i > 0$ and $\alpha > t(n-1)$, $\forall x_i \in \{0, 1\}$ for $\forall i = 1, 2, ..., n$. Note that the utility function is concave in own visits $\frac{\delta^2 U_u}{\delta_2 v_i^2} = -1$ and thus the marginal utility of v_i increases with the number of visits of other agents that i is connected to in the social network, with $\frac{\delta^2 U_i}{\delta v_i \delta v_j} = \theta$ when $g_{ij} = 1$. This means that v_i and v_j are strategic compliments when i and j are connected in the social network and each agent i selects v_i to maximize utility given the structure of the social and physical networks. We make v_i a continuous variable to represent something like the fraction of time agent i spends interacting in the center. We assume connections in the social network to be exogenous, indicating agents inherit social connections by chance of birth rather than some conscious effort. Naturally, some agents are considered "social elites" while others are considered "social outcasts" due to the relative different levels of connection, or centrality, within the social network. To measure this centrality, we do not use degree centrality (the number of direct connections a node has) or closeness centrality (the average distance between an agent and all others), but rather the Katz-Bonacich centrality measure which indicates the "weighted sum of the walks which emanate from it." This is because we want to measure both the direct and indirect connections of a node and the Katz-Bonacich measure has proven useful for this purpose in game theory applications. To represent this measure in our model, we let G^k be the kth power of G, with elements $g_{ij}^{[k]}$, were k is an integer and represents the length of the walk between connected nodes. This matrix tracks indirect connections in the network, as $g_{ij}^{[k]}$ gives the number of walks or paths between i and j in a network. In particular, $G^0 = I$ where I is the identity matrix. Thus, a matrix represented as $M = \sum_{k=0}^{+\infty} \theta^k G^k$ will have equation $m_{ij} = \sum_{k=0}^{+\infty} \theta^k g_{ij}^k$ counting the number of walks of all lengths between the nodes of agents i and j. Here, θ operates as a weighting parameter, decreasing importance of the connection as the length of the walks increases. Thus, $\theta \leq 1$ and $\theta \geq 0$. When well defined M exists, we can write $M - \theta GM = I$ and thus $M = [I - \theta G]^{-1}$. The Katz-Bonacich centrality of agent i is written as:

$$b_i(g, \theta) = \sum_{j=1}^{n} \sum_{k=0}^{+\infty} \theta^k g_{ij}^k$$

Thus, the centrality of any agent is 0 whenever the network is empty or $\theta = 0$ and is increasing and convex as θ increases. The vector of centrality measures is of the shape $(n \times 1)$ and can be written as:

$$b(g, \theta) = M1 = [I - \theta G]^{-1}$$

where **1** is an n-dimensional vector of ones. The weighted centrality measure for agent i is written as:

$$b_{\alpha_i}(g, \theta) = \sum_{j=1}^{n} \sum_{k=0}^{+\infty} \theta^k g_{ij}^k \alpha_j$$

where the weight attached to the walks from i to j is given by α_j. Where α is an n-dimensional vector, the matrix equivalent of the weighted centrality measures is:

$$b_\alpha(g, \theta) = M\alpha = [I - \theta G]^{-1}\alpha$$

Now that we have the utility function and centrality measures well-defined, the next step is to find a solution to the subutility function, that is the Nash equilibrium number of visits and resulting of interactivity. The first order condition and best-response function for agent i maximizing utility with respect to v_i is given as:

$$v_i^* = \alpha_i + \theta \sum_{j=1}^{n} g_{ij} v_j^*$$

Because the subutility function is linear quadratic, the equilibrium number of visits for agent i is linearly dependent on the visit choices of the other agents with whom i is directly connected. Thus, in matrix form, it is true that:

$$v = \alpha + \theta G v$$

Where α is a vector of shape $(n \times 1)$ containing all values of α_i. Solving for v gives us the Nash equilibrium number of visits for all agents in the form vector v^*, where:

$$(v^*) = [I - \theta G]^{-1}\alpha = M\alpha$$

For each agent i, the Nash equilibrium level of visits is given by:

$$v^*(x_i, x_{-i}, g) = \sum_{j=1}^{n} m_{ij}\alpha_j = \sum_{j=1}^{n}\sum_{k=0}^{+\infty} \theta^k g_{ij}^k \alpha_j$$

where x_{-i} is the vector of all other agents and the expression on the right is the centrality measure of agent i as defined above. The spectral radius of matrix G is given by $\rho(G)$.

Given this model, we first propose that for any given vector of geographic locations and any network G, if $\theta\rho(G) < 1$ there exists a unique interior Nash equilibrium of visit choices equal to the centrality measure for any agent i, shown as:

$$v^*(x_i, x_{-i}, g) = b_{\alpha_i}(g, \theta)$$

The equilibrium number of visits for agent i depends on its own and others' position in the social and geographic networks. This expression, $v^*(x_i, x_{-i}, g)$, increases with α and decreases with both direct commuting cost t and congestion cost $\sum_{j=1}^{n} x_j$.

We also propose, since θ is a measure of complementary in our model and we assume $\theta\rho(G) < 1$, for any network, an increase in θ will increase the equilibrium number of visits by any agent i.

Intuitively, when there are more synergies resulting from social interactions, each agent obtains more benefit from each interaction and thus will have a higher quantity of visits to the center. Additionally, as we noted before, this has the secondary effect of further increasing the number of visits as, when other agents are visiting the center more often, the marginal benefit of each agent i of visiting the center increases. However, this effect is dampened, especially for those living in the periphery, by congestion costs, which increase with the number of agents i with $x_i = 1$. Similarly, as base transportation cost t increases, the equilibrium quantity of visits decreases. Aggregating these effects, we show that:

$$v^*(1, x_{-i}, g) - v^*(0, x_{-i}, g) = -tm_{ii}\sum_{j=1}^{n} x_j \leq 0$$

since M is a non-negative matrix. Any agent for whom $m_{ii} > 0$ will visit the interaction center more when located in the center rather than the periphery.

In fact, we can show that the equilibrium number of visits for agent i is non-increasing for the increase in distance from the center of any agent. Letting x_{-ik} be the vector of locations in the physical locations for all agents except i and k such that $x_{-i} = (x_k, x_{-ik})$, we obtain:

$$v^*(1,(1,x_{-ik}),g) - v^*(0,(0,x_{-ik}),g) = -tm_{ik}\sum_{j=1}^{n} x_j \leq 0$$

Thus, letting $V^*(g)$ represent the aggregate level of visits and ergo interaction, we get:

$$V^* = \sum_{i=1}^{i=n} v^*(x_i, x_{-i}, g) = \sum_{i=1}^{i=n} b_{\alpha_i}(g,\theta)$$

Thirdly, we propose that for sufficiently small θ, aggregate interactions and the entire vector of individual interactions increase with the density of social network links and decrease as the distance of any agent from the interaction center increases. The best way to show this is via an example.

We begin with out example adjacency matrix:

$$G = \begin{bmatrix} 0\&1\&1 \\ 1\&0\&0 \\ 1\&0\&0 \end{bmatrix}$$

Now we compute the powers of this matrix, shown by:

$$G^{2k} = \begin{bmatrix} 2^k\&0\&0 \\ 0\&2^{k-1}\&2^{k-1} \\ 0\&2^{k-1}\&2^{k-1} \end{bmatrix}$$

and:

$$G^{2k+1} = \begin{bmatrix} 0\&2^k\&2^k \\ 2^k\&0\&0 \\ 2^k\&0\&0 \end{bmatrix}$$

for $k \leq 1$. It is also easily verified that:

$$M = [I - \theta G]^{-1} = \frac{1}{1-2\theta^2}\begin{bmatrix} 1\&\theta\&\theta \\ \theta\&1-\theta^2\&\theta^2 \\ \theta\&\theta^2\&1-\theta^2 \end{bmatrix}$$

We can now find the centrality measures for all three measures:

$$\begin{bmatrix} v_1^* \\ v_2^* \\ v_3^* \end{bmatrix} = \begin{bmatrix} b_{\alpha_1}(\theta,g) \\ b_{\alpha_2}(\theta,g) \\ b_{\alpha_3}(\theta,g) \end{bmatrix} = \frac{1}{1-2\theta^2}\begin{bmatrix} \alpha_1 + \theta(\alpha_2+\alpha_3) \\ \theta\alpha_1 \quad + (1-\theta^2)\alpha_2 + \theta^2\alpha_3 \\ \theta\alpha_1 \quad + \theta^2\alpha_2 + (1-\theta^2)\alpha_3 \end{bmatrix}$$

Solving this system, we must first decompose α_i for $i = (1,2,3)$. To do this, we derive the congestion cost using the equation shown earlier, assigning it to the variable s. We assign geographical locations for each agent as follows: $x_2 = x_3 = 1$ and $x_1 = 0$, thus:

$$s(g) = \sum_{j=1}^{n} x_j \; = 2$$

This means that α_i for $i = (1, 2, 3)$ is $\alpha_1 = \alpha$ and $\alpha_2 = \alpha_3 = \alpha - 2t > 0$. Now we can proceed by simplifying the system.

$$\begin{bmatrix} v_1^* \\ v_2^* \\ v_3^* \end{bmatrix} = \begin{bmatrix} b_{\alpha_1}(\theta, g) \\ b_{\alpha_2}(\theta, g) \\ b_{\alpha_3}(\theta, g) \end{bmatrix} = \frac{1}{1 - 2\theta^2} \begin{bmatrix} \alpha + 2\theta(\alpha - 2t) \\ \alpha(1 + \theta) - 2t \\ \alpha(1 + \theta) - 2t \end{bmatrix}$$

Thus, it must be true that the equilibrium level of visits to the interaction center and resulting centrality are higher for agents living in the center than those in the periphery, ceteris peribus. However, this is only true for :

$$v_1^* > v_2^* = v_3^*$$

We find the aggregate level of interaction in the network with the following equation:

$$V^* = \sum_{i=1}^{i=n} v^* = \frac{(3 + 4\theta)\alpha - 4(1 + \theta)t}{(1 - 2\theta^2)}$$

Now suppose that the social network changes, adding a connection between agents 2 and 3, while spatial locations in the geographic network remain unchanged. We thus derive:

$$\begin{bmatrix} v_1^* \\ v_2^* \\ v_3^* \end{bmatrix} = \frac{1}{1 - \theta - 2\theta^2} \begin{bmatrix} \alpha(1 + \theta) - 4t\theta \\ \alpha(1 + \theta) - 2t \\ \alpha(1 + \theta) - 2t \end{bmatrix}$$

Still, it is true that $v_1^* > v_2^* = v_3^*$. This system derives an aggregate interaction level for this new network, denoted by g_2.

$$V^*(g_2) = \frac{(3\alpha - 4t)(1 + \theta)}{1 - 2\theta^2 - \theta} > V^*(g)$$

This shows that, all else equal, a denser social network leads to a higher aggregate level of interaction and a higher quantity of visits to the interaction center.

4 Two-Stage Game

First Stage: Location Choice Here, we will introduce the concept of location choice, allowing agents to first choose their location in the network before choosing their interaction level. However the agents are aware of the subutility function. In addition, those who choose to live in the downtown must pay some cost c for living near the interaction center. This parameter is independent of the locations of other agents x_{-i} and can be thought of as the externalities associated with living in a city people travel to. As previously mentioned, there are many negative effects associated to proximity to heavy automobile traffic and a heavier tax burden resulting from paying for everyone's use of the urban infrastructure (Zhang et al.; see also IIHS; Münzel et al., 2018; Tobollik et al., 2019; Norton, 2014). In this model, the cost c is a constant for tractability, not accounting for crowding.

To begin, agents will decide in the first stage where to locate ($x = 1$ or $x = 0$). Then, after choosing a location, each agent will choose their activity level in the second stage. Given the structure of the problem, the problem must be solved backwards, beginning with the subutility function we defined and solved earlier. We also know that, so long as $\theta\rho(G) < 1$ there is a unique effort level for each individual i such that $v_i^*(x_i, x_{-i}, g) = b_{\alpha_i}(g, \theta)$. Using the best response function for v^* defined above, we derive the equilibrium utility function of agent i as follows:

$$U_i^*(v_i^*, v_{-i}^*, g) = y + \frac{1}{2}[v_i^*(x_i, x_{-i}, g)] = \frac{1}{2}[b_{\alpha_i}(g, \theta)]$$

Where equilibrium effort for agent i is $v_i^*(0, x_{-i}, g)$ when located at the *center* and $v_i^*(1, x_{-i}, g)$ when located at the *periphery.* Using this equation, we solve for the location choices of all agents. Because the weighted Katz-Bonacich centralities are a result of an endogenous equilibrium, there is a complication because the structure of the equilibrium location distribution must be known to construct an equilibrium. To solve this, we first define all agents located at the *center* as C and all agents located at the *periphery* as P.

The equilibrium utility function for an agent located at the *center* is given as:

$$U_i^*(v_i^*(0, x_{-i}, g), v_{-i}^*, g) = y + \frac{1}{2}\left[\sum_{j\in C-i}\sum_{k=0}^{+\infty}\theta^k g_{ij}^k \alpha + \sum_{j\in P-i}\sum_{k=0}^{+\infty}\theta^k g_{ij}^k(\alpha - ts) + \sum_{k=0}^{+\infty}\theta^k g_{ij}^k \alpha\right]^2 - c$$

The equilibrium utility function for an agent located at the *periphery* is given as:

$$U_i^*(v_i^*(1, x_{-i}, g), v_{-i}^*, g) = y + \frac{1}{2}\left[\sum_{j\in C-i}\sum_{k=0}^{+\infty}\theta^k g_{ij}^k \alpha + \sum_{j\in P-i}\sum_{k=0}^{+\infty}\theta^k g_{ij}^k(\alpha - ts) + \sum_{k=0}^{+\infty}\theta^k g_{ij}^k (\alpha - ts)\right]^2$$

As a result $x_i = 1$ if and only if $U_i^*(v_i^*(0, x_{-i}, g), v_{-i}^*, g) > U_i^*(v_i^*(1, x_{-i}, g), v_{-i}^*, g)$. To obtain the weighted Katz-Bonacich centrality, not including self-loops, we first denote:

$$b_\alpha^{[-ii]}(g, \theta) = \alpha \sum_{j=1, j\neq 1}^{n} m_{ij} = \alpha \sum_{j\in C-i} m_{ij} + \alpha + \alpha \sum_{j\in P-i} m_{ij} + \alpha$$

Which we use to obtain:

$$m_{ii}^2 = \frac{b_\alpha^{[-ii]}(g,\theta) - ts\sum_{j\in P-i} m_{ij}}{2\alpha - ts} + \frac{\sqrt{(b_\alpha^{[-ii]}(g,\theta) - ts\sum_{j\in P-i} m_{ij})^2 + \frac{2c}{ts}(2\alpha - ts)}}{2\alpha - ts}$$

Given this structure, there are a few conclusions we can draw. First, so long as $\theta\rho(G) < 1$, all agents i with $m_{ii} > m_{ii}^2$ will have $x_i = 0$ and those with $m_{ii} < m_{ii}^2$ will have $x_i = 1$. Thus, equilibrium location choices are represented by the following graph:

(Can I use the same graph from the original paper since it is not notably changed?)

Here, we again denote C as the set of all agents i for whom $m_{ii} > m_{ii}^2$ and P as the set of all agents i for whom $m_{ii} < m_{ii}^2$. We also define $\phi(m_{ii})$ as the relationship between position in the social network and location choice. The graph representing this model can be seen on Helsley and Zenou page 438.

Recall that the weighted Katz-Bonacich centrality measure is defined as:

$$b_{\alpha_i}(g,\theta) = (\alpha - tx_i\sum_{j=1}^{n} x_j)m_{ii} + \alpha\sum_{j\in C} m_{ij} + (\alpha - t\sum_{j=1}^{n} x_j)\sum_{j\in P} m_{ij}$$

Recall also that m_{ii} as a result of this function, captures the centrality of agent i in the social network. Helsley and Zenou show in their model that more central agents in the social network have a greater incentive to locate to the center and those with lower centrality in the social network will locate in the periphery. They conclude there will be an endogenous geographic separation by social network centrality. Our model, with the addition of linearly increasing ”congestion” charges for the number of agents in the periphery, does not change this fact. It does, however, indicate that in equilibrium, if more agents live in the periphery, the cost of visiting the center for those in the periphery will increase linearly, causing a subsequent direct effect of lowering the aggregate number of visits to the center. This will have a secondary effect of decreasing the marginal benefit for all agents, of making a visit to the center, further decreasing the number of visits to the center. Both of these effects will cause more agents to move to the periphery and make fewer visits to the interaction center until only those who are most connected to the social network, especially those highly connected to individuals residing in the center.

We now wish to focus on showing that there exists a unique subgame equilibrium. Suppose we rank all n agents by their centrality rank in the social network where agent 1 has the highest centrality measure, agent 2 the second highest, and agent n the lowest. We give each agent a type, which is defined as the agent's centrality measure. Given that two or more agents can have the same centrality, and therefore the same type, there are $w \leq n$ types in the network. When all agents reside in the *center*, we define the location function

$$\phi^C(m_{ii}) = (\alpha - t\sum_{j=1}^{n} x_j)t(2\alpha - (\alpha - t\sum_{j=1}^{n} x_j)t)(m_{ii})^2 + 2(\alpha - t\sum_{j=1}^{n} x_j)t(\alpha\sum_{j\in N-i} m_{ij})m_{ii}$$

We then use this to find an equilibrium which now means the Subgame Perfect Nash Equilibrium. In it, we find that, in any qualified equilibrium, two agents with the same centrality will reside in the same node and those with higher centrality will not live further out. We also propose that the number of equilibria is equal to one plus the number of types. We use the following steps to show this. For the following steps, assume that the number of types $w = n$. If $w < n$, then follow the steps decreasing by type, not by agent.

(i) If

$$2c < \phi^C(m_{nn})$$

there must exist an equilibrium where all agents live in the center and none live in the periphery. (ii) If

$$\phi^C(m_{nn}) < 2c < \phi^C(m_{n-1n-1} - 2(st)^2 m_{n-1n-1}$$

there must exist a unique equilibrium with $C = N - n$ and $P = n$ where N is the set of all agents in the network and $s = \sum_{i=1}^{n} x_i$.

(iii)If

$$\phi^C(m_{n-1n-1} - 2(st)^2 m_{n-1n-1} < 2c < \phi^C(m_{n-2n-2} - 2(st)^2 m_{n-2n-2}$$

there exists a unique equilibrium where $C = N - [n-1, n]$ and $P = [n-1, n]$.

(iv) Repeat the process between (ii) and (iii) until arriving at agent 1, which, again, is the agent with the highest Katz-Bonacich centrality. Thus,

(vi) If

$$\phi^C(m_{11}) - 2(st)^2 (\sum_{j \in P-1}^{n} m_{1j}) m_{11} < 2c$$

there must exist a unique equilibrium where all agents live at the periphery.

These propositions completely describe the possible states of the unique subgame perfect Nash equilibrium.

5 Pigouvian Suburbanization Tax

A Pigouvian tax is defined as a tax assessed on individuals or firms engaged in activity which is against the common good (Kagan, 2020). In essence, it aims to internalize externalities and alleviate collective action problems by increasing the direct cost of engaging in activities which lead to a less than ideal equilibrium. We propose that suburbanization is one such activity, as demonstrated above. There is an initial incentive to move away from the center as there is no crowding cost c. However, as shown above, this move will lead to a lower aggregate interaction level, leading more agents to move to the periphery as the marginal benefit from interacting, and therefore of living in the center, decreases. This creates a cyclical process whereby, for sufficiently small values of c and a sufficiently dense social network, lead to a Nash equilibrium which is less than the aggregate optimal. This is because, when deciding where to live and therefore their subsequent level of interaction, agents are strictly considering their own marginal benefit from each interaction, not the total benefit, which is the individual marginal benefit plus the additional benefit to all other players to whom she is connected in the social network. Thus, the aggregate level of interaction will be less than in a Pareto optimal case. We can think of this scenario as analogous to suburbanization, where families choose an initial benefit of moving to a suburb, but the aggregate level of social capital decreases due to the resulting increase in interaction costs. This is one potential explanation for the supposed decline in social capital in the United States in the second half of the twentieth century, according to Putnam's *Bowling Alone*. To increase aggregate utility, we propose a tax on those living in the periphery, or suburb.

To determine the ideal Pigouvian tax, we would need to know the entire social network structure and the utility function for every agent. With this knowledge, we could optimize for maximum aggregate utility given a particular social network. Because the exact size of the externality for each visit by an agent to the center is dependent on the agent's connectedness in the social network, the corresponding tax will also depend on the exact makeup of the social network. Such information is, for all practical purposes, unavailable to the social planner, making an exact optimization effectively impossible. Thus, while exact Pigouvian optimization is effectively impossible, we can still construct our second best option: a tax which, while not optimal, still allows us to achieve a universally better equilibrium in the model.

To do this, we must consider the source cause of the externality. Each additional person in the social network living in the center creates a positive externaliy, as it increases the number of visits said agent will make to the interaction center, increasing the marginal utility of making a trip to the interaction center for all socially connected agents. Conversely, moving to the periphery has the exact opposite effect, creating a negative feedback loop wherein the marginal utility of visiting the center decreases with each additional person moving to the periphery, pushing more agents to locate to the periphery and so on. Additionally, there is another negative externality of living in the periphery in the form of congestion. As each additional agent moves to the periphery, the marginal utility of visiting the interaction center for all peripheral agents decreases as congestion costs increase linearly with the number of agents in the periphery, further reinforcing the negative feedback loop. Given that the source of the inefficiency is too few visits to the center resulting from too many agents choosing to locate in the periphery, any tax aiming to create more optimal equilibria should encourage more people to locate to the center rather than the periphery and internalize the external loss of utility resulting from an agent's decision to live in the periphery rather than the center.

Any agent i will choose to locate in the periphery if the crowding cost of living in the center, denoted by c, is greater than the lost utility for i resulting from fewer visits to the interaction center. As we have shown above, any agent living in the periphery will be less socially connected than those who choose to live in the center and an agent will always have fewer visits to the interaction center when they live in the periphery when compared to the center. Given that we do not want to penalize further the visits those who live at the periphery do make, we are left with the choice of somehow penalizing agents for locating in the periphery and benefit those who live in the periphery.

We propose, as a practical, tractable solution, to effectively institute a tax on living in the periphery dependent on the number of agents in the network, and use any revenue it generates to subsidize those living in the center by reducing crowding cost c. Thus, we can ascribe to those living in the periphery a tax equivalent to $\frac{c\sum_{i=1}^{n} x_i}{n}$ and the cost is then used to subsidize living in the center, with the crowding cost decreased by an equal amount. This changes the crowding cost from c to $\frac{c(n-\sum_{i=1}^{n} x_i)}{n}$, a decrease in any case where at least one agent lives in the periphery and no change when every agent is in the periphery. This tax-subsidy combination allows the government a balanced budget while decreasing the motivation for agents to move to the periphery, leading to a universally better aggregate utility and alleviating, albeit imperfectly, the collective action problem.

In comparing this model to suburbanization, we can think of this process of attaching a progressively increasing tax on those who choose to move to the periphery not necessarily in terms of an explicit tax on those who choose to live in suburbia, but rather a decrease in the already existent subsidising of suburban development. Federal and local governments in the United States effectively subsidize sprawl via fossil fuel subsidies, strict parking minimums, reduced tax income per hectare, and significantly infrastructure spending per taxpayer among a litany of other methods (Tiecher et al., 2021). These funds have historically come from debt or the far more financially sustainable, denser developments within a jurisdiction. We can think of this new tax-subsidy dynamic as its inverse and the reversal of a long-standing practice. That is: reduce subsidies to those living in suburbia and allow those funds to remain within the denser developments of the cities. In effect, the government should no longer subsidize a bad equilibrium.

6 Conclusion

Suburbanization is a process which radically changed the way in which millions if not billions of people have lived their lives over the past century. Thus, we found modeling such a process, especially in regard to how it affects interpersonal interaction and social capital, highly important. Basing our model on that of Helsley and Zenou, we constructed a dynamic two stage game representing such a process. Using a game theory framework, we constructed a model such that agents could first choose their location in a *periphery* or *center*, analogous to a city and its suburbs. Then, agents could choose their level of interaction, or the number of times they visit the center. Given an exogenous social network, agents' benefits from each interaction are determined by the number and interaction level of those with whom they are connected in the network and marginal cost determined by location choice and congestion. We proved many characteristics about this network, including that denser social networks lead to higher levels of interaction and utility, those living in the *center* will have higher levels of interaction than their peripheral peers, and that those more connected in the social network will always live closer to the interaction center than those who are less connected. Finally, we showed that such a network will lead to a less than socially optimal level of interaction and introduced a Pigouvian congestion tax which would create a better equilibrium.

The implications for our model on public policy are highly dependent on the level of truth one ascribes to Putnam's theory of declining social capital. It is also highly dependent on the assumptions underlying our model. While we may believe these assumptions to be reasonable, an empirical analysis applying the model to real-world data. We have proposed a mechanism explaining previously observed phenomena, but that does not mean that it cannot be tested. Perhaps an empirical study examining the relationship between ease of interaction and number of interactions, as well as the relationship between suburbanization and social interaction. One potential difficulty in analyzing the truth of our model, however, is that recreating alternate conditions is practically impossible, since people's actions are inextricably linked to their social network and the decisions of others. To circumvent this, perhaps constructing a stochastic, repeated version of our model in a digital simulation and comparing its results with real world trends would further validate this thesis. To conclude, we have introduced a compelling new model to explain a complicated social phenomenon which, we hope, will help inform future public policy decisions to create a better world for everybody.

7 Works Cited

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